

Drawing creating space

JOHN STELL

Some recent drawings are discussed in which the author recorded the passage of time in a familiar domestic space. Initially expecting to represent the space using ideas from different kinds of geometry, it is found that the drawings do not fit into this framework. Alternative conceptualisations are that the drawings record a process of exploration of the space, or that drawing is creating a space that only exists by virtue of the drawing process itself. The interactions between these options support reflections on how perception and spatial experience is mediated through drawing.

Keywords: spatial experience, drawing, tracing, time, architectural space

Discutem-se desenhos recentes em que o autor registou a passagem do tempo num espaço doméstico que lhe é familiar. Inicialmente, a expectativa era a de representar o espaço com recurso a ideias de diversos tipos de geometria; concluiu-se que os desenhos não se coadunam a este enquadramento. Concetualizações alternativas indicam que os desenhos registam um processo de exploração do espaço ou que o desenho cria um espaço que apenas existe em virtude do próprio processo de desenho. As interações entre estas hipóteses dão base a reflexões sobre como a percepção e a experiência do espaço são mediadas através do desenho.

Palavras-chave: experiência espacial, desenho, traçado, tempo, espaço arquitetónico

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THE SPACE IN THE ROOM

I am in a familiar domestic space. A room, roughly square and maybe almost 5m on each side. There are windows in two opposite walls, and the other pair of parallel walls has doorways in diagonally opposite corners. There is a staircase in a third corner, where once a door led in from the outside when the original building was sub-divided into a terrace of small houses. Once the window in the rear wall was a doorway leading to a lean-to extension of which only the walls remain outside. This doorway itself had earlier been a window, and the precise sequence of alterations and interventions is largely undocumented, but the traces left are in general clearly readable.

It is a space I have known for almost 60 years, although there are no clear recollections from the earliest part of that time. The building itself only escaped demolition through the intervention of my father, who must have meticulously recorded it in measurements in feet and inches as well as in photographs that are archived but have not been studied or seen for years. While not somewhere I lived until relatively recently in the span of 60 years, it carries memories. Given the doorways and the staircase it has more of a transitional feel than a room to settle in. Often called “the middle room”, naming it in relation to other spaces rather than its intrinsic qualities, it was often a route from one place to another rather than a destination.

I am facing the challenge of how to draw this space, or even simply how to draw in the space. Is there, maybe, a toolkit of ways of representing space that can be applied here? How can anything new be conjured out of such a familiar space?

REPRESENTATIONS OF SPACE

One obvious approach is the conventional plotting into space of coordinates. Taking numerical measurements, the room can be reduced to several polygonal surfaces joined along specified lines. Such things are routinely now processed by software that allows us to derive three-dimensional virtual reality visualisations, as well as conventional plans, sections, elevations, perspective renderings and other projections such as isometric, axonometric, etc. The process of gathering the numerical coordinates yields the room as a surface, approximately a rectangular box. Solidity is reduced to the interior faces of the walls, the floor, and the ceiling. The thick stone walls are indicated without substance.

This seems an unpromising direction. Surely such a conventional process would be incapable of

revealing anything new about my understanding of the space as part of my experience. Several authors have questioned the relationship between this kind of representation of space and actual human experience. For example, Whitehead (1929, p.v.) who in *Process and Reality* stated: ‘In this enquiry we are concerned with geometry as a physical science. How is space rooted in experience?’ This geometry as a physical science was to be something different from the conventional geometry of Euclid. The connection between perception and geometry was also considered in need of investigation by Felix Klein, one of the most notable geometers of the 19th century.

Psychologists, by the way, now generally distinguish (following the example of E.H. Weber) visual space, tactile space, motor space as perceptual substrates. But how do they combine themselves into ‘geometrical space?’ Associations certainly play a role, but are they enough for an explanation? E.H. Weber also distinguished a general spatial sense. (1909, p. 48)

More dissatisfaction with the ability of coordinate geometry to model the “real world” in assuming that measurements could be made to arbitrary degrees of precision, is shown by Poston (1971a,b). Poston argued for a “fuzzy geometry” in which points which were close enough together would be indistinguishable. This applied to time as well as to space: “*a film builds visually continuous motion out of visually indistinguishable pictures at visually indistinguishable moments*” (Poston 1971b, p. 28). Poston’s geometry is mathematically very different from the conventional account of space, but it appears to offer little insight into the representation of space through drawing. When making measurements in practical everyday situations there may not be an explicit tolerance or limit of distinguishability, but measurements are made to the nearest centimetre or whatever is appropriate. The view of fuzzy geometry, also called tolerance geometry, fits how we behave.

Despite the many dismissals of conventional geometry as capturing experienced space, describing the room in terms of cartesian coordinates is a somewhat surprising exercise to carry out in the traditional way by hand. Imagining at first that this must be simply reducing the room to a collection of numerical coordinates, you soon realize that there is a large gap between simply knowing the coordinate points themselves and the knowledge of how they relate to each other. It is necessary to know which lines between points and which surfaces bounded by lines, are actually part of the model of the room. Even as a thought-experiment, or an imagined

drawing, the process is very instructive in terms of experience. Adopting the traditional tools of tape measure and notebook, I have to physically move along the edges of the room. There are physical challenges to completing the task alone, especially in anchoring one end of the measuring tape. The distances themselves are recorded in a specific sequence that follows a sketch plan. Not only the edges are followed with the measure; there are also invisible distances between the opposite corners that are needed to be sure the shape has been grasped. The plan is not all. The lack of true verticals in the walls means more careful use of the plumb line, ladders and recording.

The numerical model at the end of the measuring process holds little trace of the physical experience of the space. However, we can consider the process not as a means to an end, not as a way to obtain the all-important numbers, but as a performance where the idea of gathering measurements provides directions. This viewpoint, including the importance of moving through the space to make measurements, reminds us that human experience of movement in space and Euclidean geometry are by no means unrelated. Ivins has a footnote that

If one remembers correctly, it was Ernst Mach who picturesquely pointed out that if [humans] were fastened immovably to rocks like molluscs in the sea they could have no sensory intuition of Euclidean space. (1938, p. 8)

This forms part of Ivins' argument that Euclidean space derives from movement and the tactile environment, but the projective geometry that supports perspective descriptions is the result of visual experience. The particular case of parallel lines is significant here. The tactile experience of moving one's hand over parallel edges on a piece of cut wood supports the idea of parallel lines never meeting. However, the visual experience of the edges of straight road meeting in the distance leads to another geometry.

Thinking of taking conventional measurements as a process in this way brings in experience but seems likely to produce drawings that are more conceptual rather than having a visual relationship to the experience. I reject the idea of carrying out this process in detail and consider if there are other notions of space that might be more productive. Whitehead's search for a "geometry of experience" did lead to a different way of thinking about space and its computational representation. Might this provide a way of drawing the space that can reveal something about perception? Looking for geometry based on experience, Whitehead rejected the

conventional infinitely small points as the building blocks of the theory. Points have no physical size; they cannot be perceived even in theory. Whitehead (1929) builds a theory of space and time on the idea of "extensive connection". Spatial perception can detect regions which extend in space but not infinitesimal points. These regions are the building blocks of Whitehead's account, and two regions may have a relationship of "connection" to each other. Thinking of regions in a simple case as two dimensional shapes that can be drawn on a flat surface, connection would correspond to the shapes overlapping or touching on the boundary. This approach has developed into a representation technique applied in Artificial Intelligence (Cohn and Renz, 2008) where spatial relationships between entities are central. Such relationships may be encoded in terms of a primitive notion of connection and can include various kinds of "inside", "through", "between", etc.

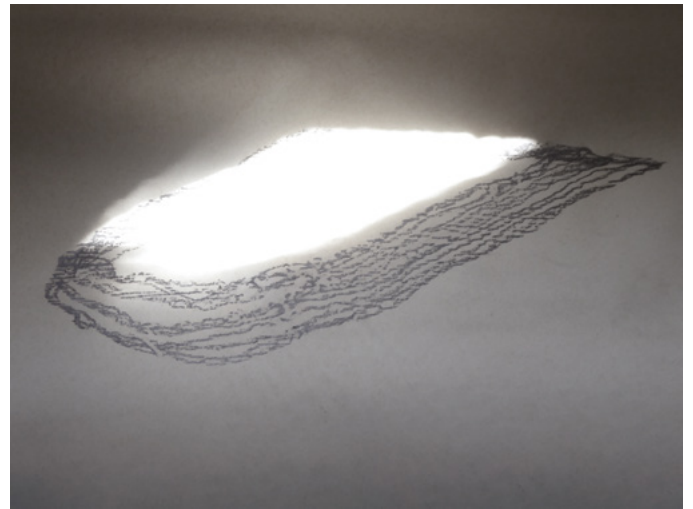
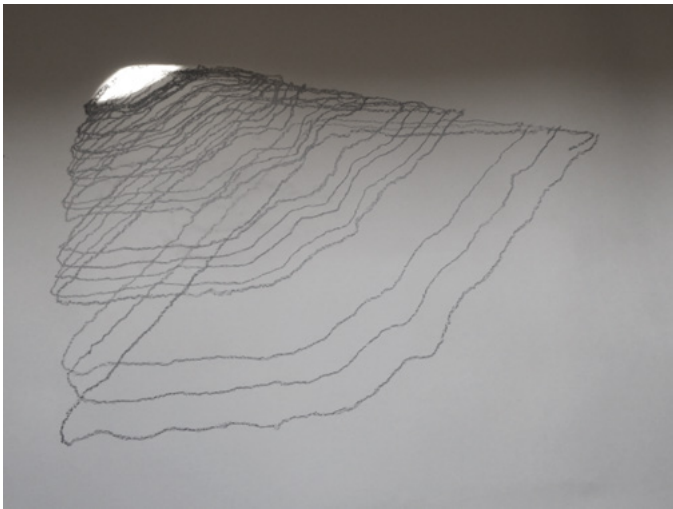
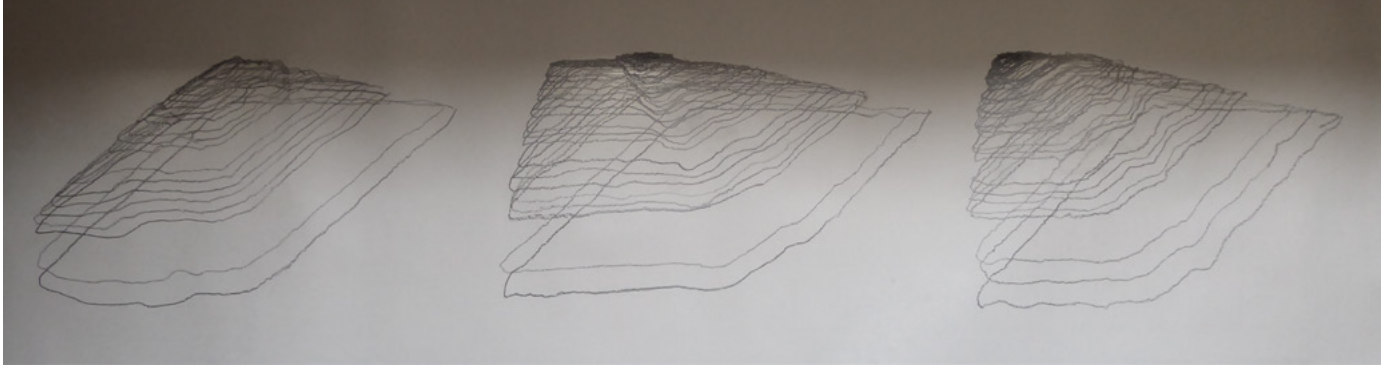
I consider how these relationships would be drawn. It is possible to build up a network of entities linked by labelled arcs for the relationships between them. Just as with the imagined exercise of the process of gathering detailed measurements, the drawings that would result do not seem to be essentially visual. That is, they carry no content beyond what is expressible in text. I can write that "the diagonally opposite corners have doors" without losing anything from making a diagram containing labelled blobs for the entities (corners and doors) and drawing arrows labelled by "diagonally opposite" and "is in". This can be a drawing, but I am looking for something essentially visual and not purely conceptual. I try a different approach.

THE FIRST DRAWINGS

A large piece of paper is unrolled onto the floor of the room. The floor is clear of furniture and the paper sits on the stone flags trying to regain its rolled up form. Sunlight comes through the windows on one side. The shape of each of the four openings appears as a bright area on the paper. I draw around the perimeter of each of the bright areas with a stick of 9B graphite. Sometimes it catches on imperfections in the underlying floor; sometimes the boundary between dark and light is not straight. The glass has irregularities that disturb the outline and the sunlight is sometimes delimited not by the metal edges of the frame but by striking it at such an angle that it is caught by the stone mullions dividing the parts of the window.

I try to work at a regular pace, drawing once around the first outline then moving on to the next and eventually coming back to the first one after

Fig. 1 Drawing from first series showing pyramidal forms (Photograph of drawing in situ, about 150 x 42cm, John Stell, 2022).



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Fig. 2 Drawings from first series showing layers of light (Photographs of drawing in situ, John Stell, 2022).

completing the last. By the time I return to the first outline the changed position of the Sun is quite apparent. I am now starting to draw the first outline in a different position. The effect was actually just visible even after drawing the first outline once; now the drawing of the other three having intervened, the effect is marked. The second tracing of each of the openings is displaced and also has a modified shape. As the Sun continues to climb higher, the bright areas on the paper shrink in height. The shrinking in height introduces a new feature. Making an effort to trace at approximately the same rate all the time, I find that the shrinking height of the shapes means the perimeters become shorter. Each one is completed in less time as the drawing goes on. Keeping to the process of drawing each one in succession I notice the traces of each become closer and closer. The interval in which it had to wait its turn to be recorded becomes less and less.

Eventually the bright areas shrink to nothing at all. The drawing has reached its natural conclusion. Unexpected aspects of the stonework around the windows are evident. The rhythmic process of repeatedly drawing in the same clockwise fashion has encouraged the paper to twist slightly. Even weighed down to prevent it rolling back up, it has not remained quite fixed with respect to the floor. What is most striking is the sense that the drawing presents a three-dimensional form. Something pyramidal has emerged out of the movements, Fig. 1.

The light through the piercings in the boundary of the room must, I realize, be a kind of prism roughly rectangular in section and bounded horizontally by the floor and vertically by the window. Maybe this volume might be revealed through dust or smoke if the room as a whole could be sufficiently darkened. But the volume appearing in the representation on the paper does not appear to be this itself; it seems to be some form created through the changing shape of just one face of this prismatic solid as becomes more and more skewed, shrinking until the volume it bounds collapses into flatness when the Sun's rays become parallel to the outside wall of the room.

The impression of light somehow describing a pyramid seems similar to Anthony McCall's *Solid Light* films from the 1970s. In my drawings the lighted area is a two-dimensional area shrinking over time but generating a three dimensional solid, whereas McCall's most well-known *Line Describing a Cone* (1973) appears to the audience as a one-dimensional line sweeping out the two-dimensional surface of cone, but not the interior volume of the cone. In *Four Projected Movements* (1975) the four installation drawings (Kelly 2013, pp.188-189) show a projected line sweeping out the quadrant base of

a quarter cone so that a three-dimensional solid is described. This is still different from what seems to be shown in my drawing, which would be more like the base of a cone rising up to the apex as it moves through the volume. Anne Wagner (2013, p. 17) writes about how McCall's films originate in drawings, a white line on a black background for each frame. Thinking about my first drawings, the process is reversed: the light comes before the drawing and leaves black traces on a white background. There appeared to be a volume depicted in the drawings, but it is initially elusive; I was only able to make sense of what it was when I saw that there was a new way to think of the space in the room, Fig. 2.

A FRESH SENSE OF SPACE

It was only after making the first drawings that a new aspect of the space became evident. This way of thinking of the room is not in the least novel or, in retrospect, surprising. It was simply a way of relating to the passage of time in the room that had never occurred to me as a vivid experience as opposed to a consequence of well-known facts. The interesting aspect is not the realisation itself, but the question of how this realisation came about, and whether there might be some mechanism that could be relied on to produce such new viewpoints – rather like a method in the same way that conventional perspective arises from a method and way of understanding space?

I had started imagining the room as a static space, within which people moved and outside which the rest of the world had carried on for at least two centuries and probably closer to three. I was in the room and the Sun moved outside the window. At some point this view flipped quite suddenly and I thought of the room moving, being tilted at different angles to the Sun as the Earth rotated each day and progressed on its annual orbit. Instead of the paper recording a moving event as a window through which movement is observed, the paper itself was moving. As time went on the paper was oriented at changing angles to the light from the Sun. The paper was moving with the whole room and the changing position of the light on the paper was caused by the window meeting the parallel beams of sunlight at differing angles.

We are accustomed to thinking of the Earth moving around the fixed Sun, while also often thinking as if the Earth is stationary. For our common-sense everyday experience, we can interchange between the two views and, outside the realms of physics and astronomy which are invisible to everyday experience, these are two interchangeable views. The idea

of relative motion gives us the idea than one of these views is as good a way of thinking as the other. But we have no everyday way of distinguishing them by experience, although scientific experiments are another matter. This indistinguishability appears in a reported question of Wittgenstein:

He once greeted me with the question: Why do people say that it was natural to think that the sun went round the earth rather than that the earth turned on its axis?' I replied: 'I suppose, because it looked as if the sun went round the earth.' 'Well,' he asked, 'what would it have looked like if it had looked as if the earth turned on its axis?' (Anscombe, 1959, p. 151)

Seeing that it was possible to conceive of the paper as moving and the Sun as fixed enabled me to understand what the pyramidal volumes described earlier represented. Somehow the floor was tilting towards where the windows were. The precise motion due to the rotation of the Earth is not easy to envisage and is more complex than turning on an axis along where the window wall meets the floor. However, as a very crude model, turning in this way is easy to imagine and does have the correct effect that as time passes the floor moves towards where the windows used to be. A quick experiment with a paper substitute for the wall with windows and floor taped to a roughly hemispherical mixing bowl which is then rotated provides a check that this picture captures the most basic features.

On the floor the bright area of each window changes over time in shape but also is moving upwards so it must sit above the earlier, larger, illuminated areas. The roughly pyramidal volume made by this sequence of shapes, this succession of diminishing layers, can be imagined as rising above the largest area, the one drawn first in the drawing. This can be seen as a volume of light, and, in comparison with the Solid Light films of Anthony McCall mentioned above, the projection is onto a surface moving towards the projector but at a constantly changing angle. There is no direct visual illusion of the volume itself, but the projection on the paper of the slices through allows us to imagine such a volume. The pyramids grow as if each circuit of the outline belongs on a separate video frame. If able to draw fast enough two adjacent frames would appear visually the same, so we are back with Poston's fuzzy geometry:

Similar considerations apply to measuring short intervals of time as with short distances of space, and the idea of time as a fuzzy continuum is equally well motivated by perception; just as a newspaper photo builds a visually continuous line out of visually indistinguishable dots at one moment,

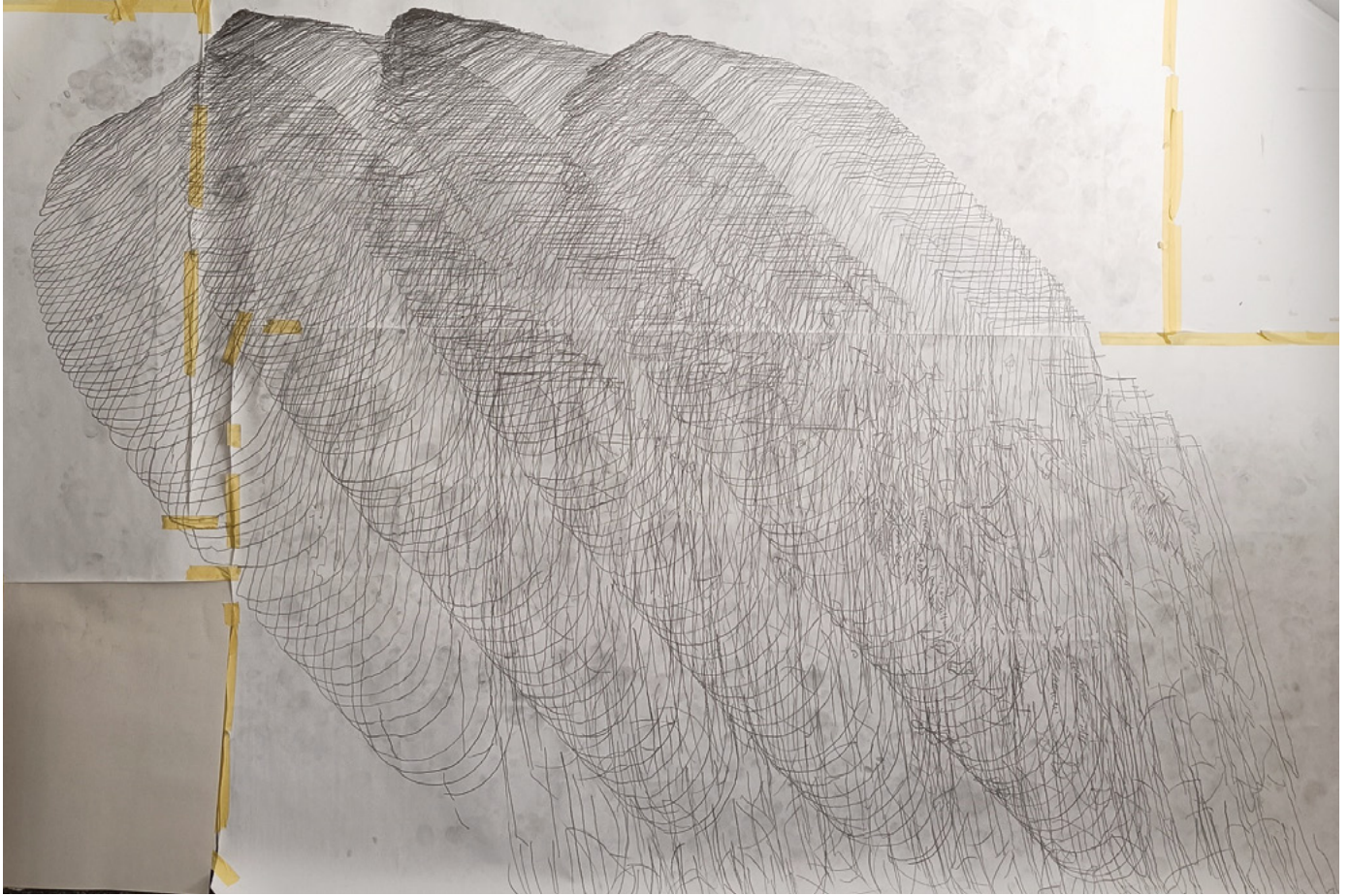
a film builds visually continuous motion out of visually indistinguishable pictures at visually indistinguishable moments. (Poston, 1971b, p. 28)

SECOND DRAWINGS

I start a second series of drawings. It is earlier in the morning and the sun is only beginning to outline the windows on the floor. This is much larger, about 2m by 4m. I work as before in completing each window outline at a regular pace before moving on to the next one. Starting earlier means that the sunlight is obscured by trees and a plant immediately outside the window casts a shadow on the paper. The outlines of the windows are not sharp. The way the light filters through the trees is combining with the double-glazing to conjure multiple edges for the windows. Later in the day, as the sunlight becomes stronger and the trees are out of the way, this effect vanishes. But to start with, the process of just drawing the window edges is no longer simple. I decide to draw the multiple edges and to include the shadow of the large teasel that has established itself outside the window. The spiky seed heads create distinctive marks. The combination of gentle winds and light through the moving trees means that these marks are only rough indications of where things were. In the early stages the drawn shapes change rapidly from one iteration to the next, Fig. 3.

The whole drawing is an almost continuous process of about four and a half hours. It is physically tiring and the end result bears traces of the process of having to crawl and walk over the paper. On viewing the drawing in a vertical setting (Fig. 3) two features stand out. One is the way that the repeated drawing of the window boundaries rather obscures the sense of movement – having so many closely drawn images negates the effect of change rather than enhances it. The other feature is that there is an area (Fig. 4) where the indications of the teasel dominate and the later window edges have created a framework that appears to contain the plant. It is like a greenhouse with the window tracery caging the plants inside.

This framework containing the outside prompts another change in how the room can be conceptualised. The room is still tipping up as the Earth rotates, but it is no longer a solid, cube-like form moving through space. The successive positions of the windows and edges of walls trace out a moving cage. From the way the light changes, the earlier position of the plant (with respect to a fixed Sun) is later a position actually inside the room. It is possible to see the room like a carriage in a train. A person re-



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Fig. 3 Second series, overall view of first drawing (Graphite on multiple paper sheets, 2390x3560 cm, John Stell, 2022)

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Fig. 4 Second series,
drawing in progress
and detail from Fig. 3
(John Stell, 2022).



mains seated in a train but occupies a succession of locations with respect to the track which are all, at different times, locations in every different carriage. The room traces out a passageway, a tunnel in space and time that intersects with the form swept out by the plant as it moves. This is not in the least mysterious or novel; two people can follow the same road at different times but never meet. However, thinking of the room in this way feels very different from the idea of a fixed space which contains things. The walls have dissolved and the distinction between inside and outside becomes unclear in the new perspective.

THE THIRD DRAWINGS

The walls have lost their solidity with the second viewpoint. The drawing itself however seems too solid. The process of drawing continually has given such a closely packed framework that it appears to be woven densely. Exploring how to move away from this, I adopt the approach of drawing only every 15 minutes. Each quarter of an hour I go through the same process as in the previous drawing. I draw round the outlines, including the teasel standing outside and indicate some of the uncertainty in the outlines in the earlier part of the morning. The visual contrast is not especially striking. The framework of lines is more open, especially toward the end. The way the shape of the window changes is somewhat more pronounced. However, the experience of making the drawing is quite different, even though the drawing itself does not seem to convey this. Instead of drawing the four windows in order and immediately repeating the process, I have some 12 or 13 minutes each time to do something else. There is a clear experience of a succession of discrete drawings being made. Time is experienced as a series of snapshots rather than as a continuing duration, Fig. 5.

This time the drawing does not evoke a new perception of the room in the same way as before. By leaving the room and focussing attention on something else between each episode I am struck by the way the space is separate from my experience of it. In the earlier drawings I had a sense of recording something like an ongoing event that I was a part of. By sampling the state of the room at discrete times, I experience the room in a more separate way. It appears to have its own trajectory which I am no longer carried along with. This links back to the approach of Whitehead using extensive connection between regions of space, and more generally of space-time. Whitehead (1925, p.75) writes about the importance of events in his theory giving an example: “the event which is the passage of the car is a part of the whole

life of the street”. The events which are the continued existence of the room and my own life have relationships that are more complex than one being a part of the other. In a sense, they intersect with each other, Fig. 6.

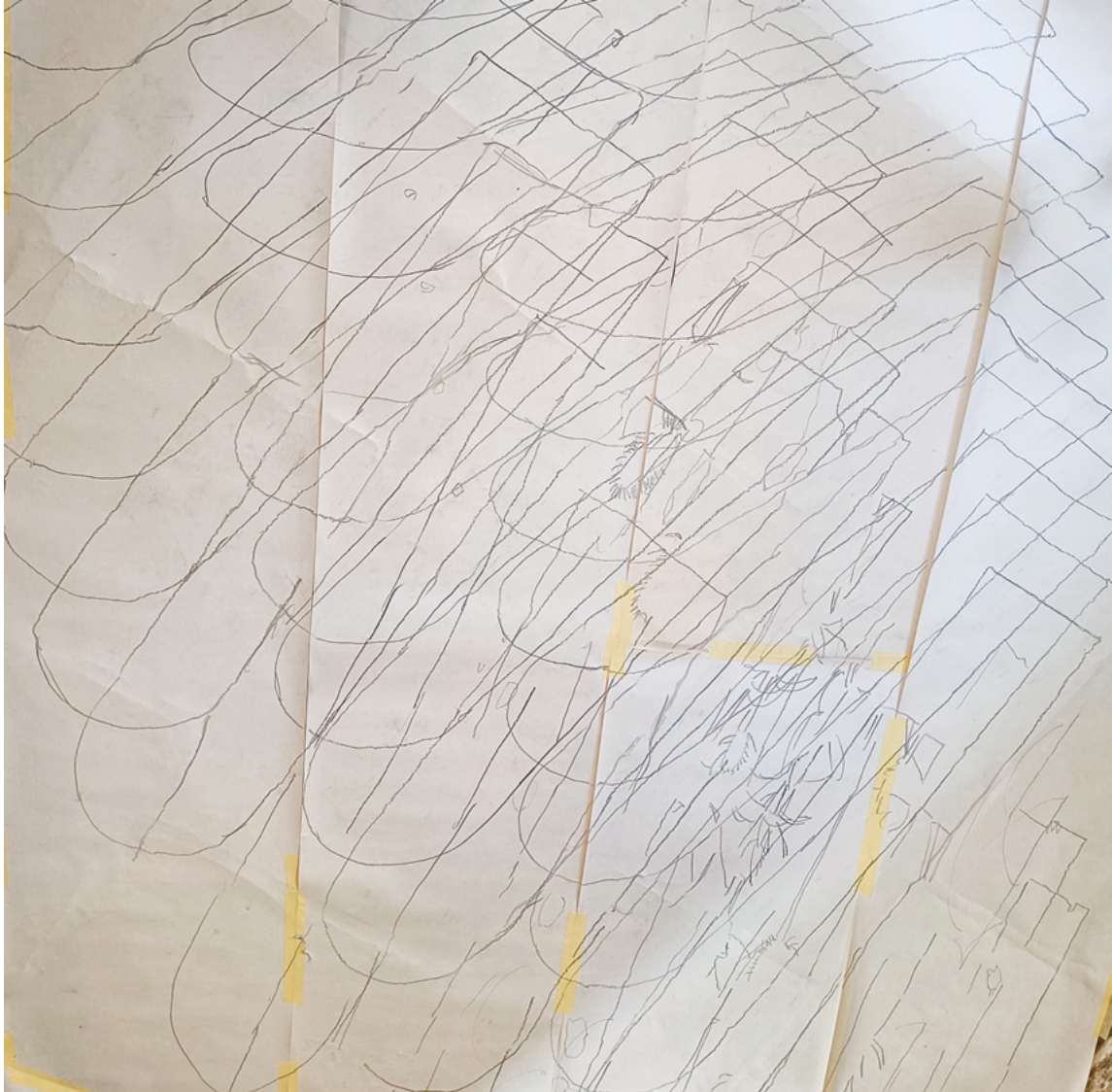
The idea of intersecting trajectories recalls some much earlier drawings (Fig. 6) made with an harmonograph (Goold n.d., Ashton, 2003). The drawing in Figure 6 was made with a device having two pendulums that could swing in elliptical orbits. One carried a board with paper, the other had an arm with a pen. This form of mechanised drawing has both the pen and the paper moving separately. The combination of the two motions constructs the drawing. Initially, the relevance of the harmonograph seems to be the continuous trace it displays, and the way this can be imagined as the progressive motion of the room throughout the drawing. In the first and second series of drawings I can see I was following the trace, being in the moving room, all the time. In the third series, I was sampling the trace at discrete intervals while on another trajectory that cut across, backwards and forwards.

The harmonograph has another and more telling relevance; one that applies not only to the third series of drawings. Both the harmonograph and the drawing process I explored are made by a combination of the paper moving and the drawing instrument moving in a separate motion relative to the paper. With the harmonograph, the pen has a determinate path that is effectively independent of the paper. In the drawings described here, the motion of the Earth carries the paper and me along with it. But the trace left by the graphite is made by a separate motion, driven by the way my hand moves and my perception of the position of the boundary of the window outlined on the paper. This suggests further experiments, and simply drawing round the outline of just one of the four windows without stopping to switch to the next window, does indeed produce a single line that repeatedly intersects itself very like the trace of an harmonograph.

REFLECTION ON THE PROCESS OF DRAWING

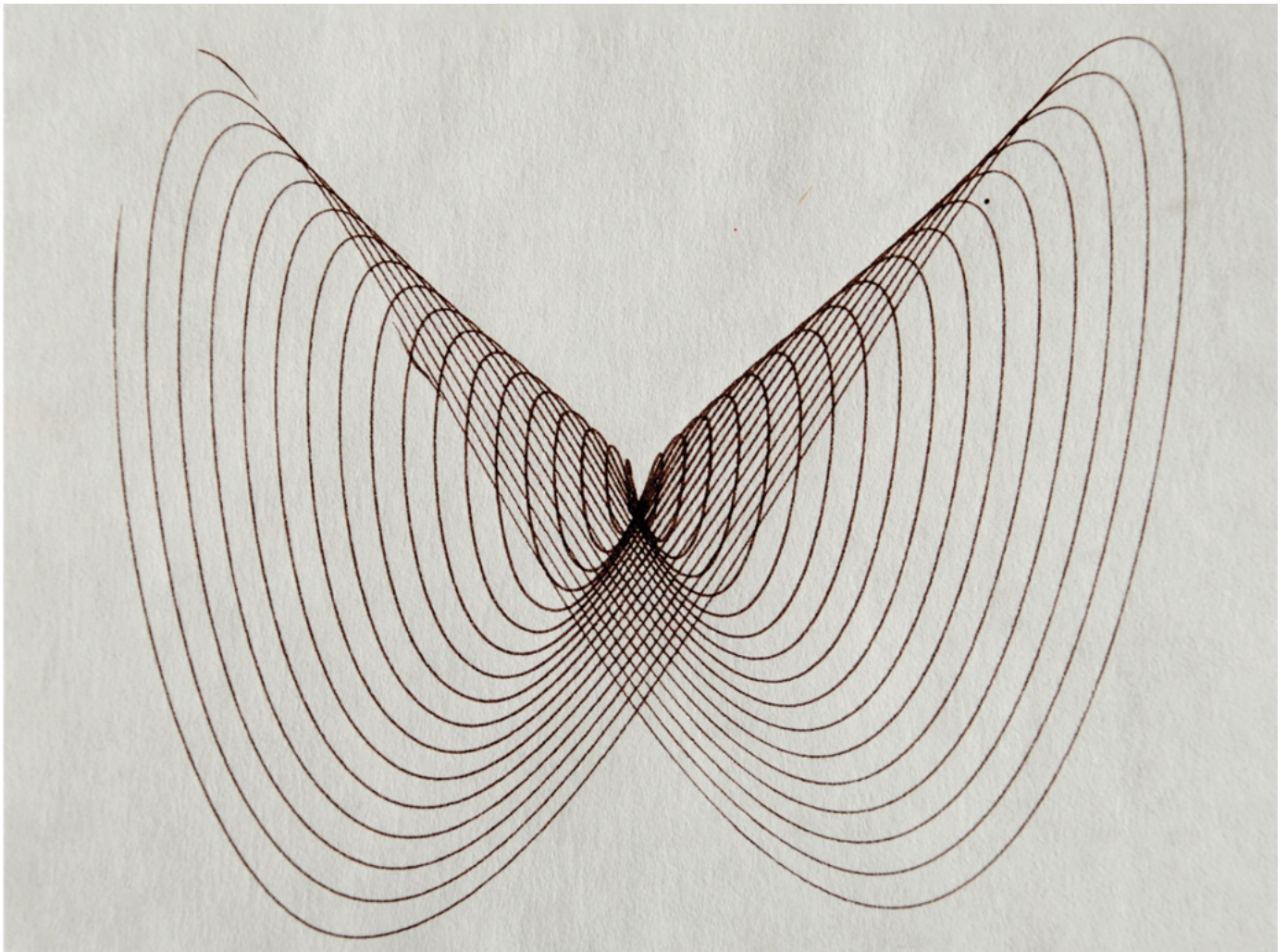
The drawings discussed here have been made by a very traditional technology of using graphite on paper. To step back and consider what the conclusions are from this activity it is instructive to consider a view from one of the pioneers of a very different technology in the context of space. Char Davies (2004, p. 103) emphasises that virtual reality’s “perceptually refreshing potential is possible *only* to the extent that the virtual environment is designed to be *unlike* those

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Fig. 5 Detail from
third series (John
Stell, 2022)



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Fig. 6 Harmonograph drawing, John Stell c1975.

Fig. 7 Unfocussed effects of light on floor and through a glass of water. (Photographs documenting drawing process, John Stell, 2022)



of our everyday experience". She argues that "immersive virtual space can be used to convey alternative sensibilities and worldviews" only when these spaces are "constructed in ways that circumvent or subverts the medium's conventions". This is arguing against the idea that virtual reality should aim to imitate our real-world experience as closely as possible. Such realism is achieved through the modelling of virtual space as conventional three-dimensional coordinate space. That is, the kind of space considered at the start of this essay as a way of modelling the space of the room that has been the main character here. Through means such as navigation and movement controlled by the breath and the changes to the body's centre of gravity, Char Davies manipulated virtual space to "provide a means of perceiving freshly". She showed it was possible to "redirect attention from our usual distractions and assumptions to the sensations of our own condition as briefly embodied sentient beings immersed in the flow of life through space and time" (Davies, 2004, p. 71).

The process described in this essay is an example of using drawing to explore part of space and time only on an individual level, instead of building an environment for multiple participants as in Davies' work. This is a very different activity and comparison of the aims or significance is not intended. However, in seeking a way of understanding how the drawing experiences fitted together, the phrase "briefly embodied sentient beings immersed in the flow of life through space and time" stood out as important. The drawing project had started without any specific research questions, except the very vague one of "what can be made out of the experience of this particular space through the activity of drawing?". As recounted above, a process evolved in which the experience of the room as solid and fixed developed into a much less conventional perception. The physical process of making large drawings was important in understanding how the new perception had emerged through bodily activity. Virtual reality (VR) has advanced in the quarter century or so since some of Davies' work, and continues to be subverted by artists to reveal non-virtual realities. The process used as a means of interrogation described here suggests, to use words originally applied to VR in (Davies, 2004, p. 103), that the technologically primitive activity of drawing still has much to offer as "a perceptually and conceptually invigorating philosophical tool".

The tool of making drawings had had a dramatic effect. The solid and familiar room vanished. Pyramids of golden light erupted from the floor. A greenhouse was flying through space with plants that were alive with independent movement. My

own activity was sometimes in this space, travelling along with it, and sometimes cutting across it, like a passenger stepping in and out of a paternoster lift, or jumping between moving platforms like a character in a video game. Wielding the tool had somehow brought all this about. It was also clear that the drawings were only the initial steps and they could all be developed in further ways, Fig. 7.

The starting point appeared to come out of nowhere. Before it occurred to me how to start drawing, I saw a patch of sunlight on the floor; I looked at it from an adjacent room. I noticed what happened when I took photographs of the light through a glass of water (Fig. 7). There seems to be no explanation of where fruitful ideas come from, but maybe thinking of the boundary as a place of particular significance had helped here. The initial expectation and the first drawings are quite at variance. I understood that the light would move on the paper, but I had only a rough idea of quite what speed would be involved. The overall change of perception was completely unexpected.

Drawing can function in many different ways. The drawings described here were not using drawing as a way of representing things, either physical or imagined. It was used as a tool that allows us to go on a journey of exploration in which familiar things are re-created in surprising ways.

REFERENCES

- Anscombe, G.E.M. (1959) *An introduction to Wittgenstein's Tractatus*.
 Ashton, A. (2003) *Harmonograph: A Visual Guide to the Mathematics of Music*. Wooden Books.
 Cohn, A. G. and Renz, J. (2008) "Qualitative Spatial Representation and Reasoning." Eds. F. van Harmelen, V. Lifschitz and B. Porter. In *Handbook of Knowledge Representation*. Amsterdam: Elsevier, pp. 551-596.
 Davies, C. (2004) "Virtual Space". In Penz, F., Radick, G., Howell, R. (eds) *SPACE In science, Art and Society*, Cambridge University Press. Chapter 4, pp. 69-104.
 Felix Klein (1909), Seminar on the Psychological Foundations of Mathematics Translated by Eugene Chislenko (draft) [accessed 24 July 2022] <https://www.math.nyu.edu/~tschinke/books/klein/klein.pdf>
 Goold, J. et al (n.d) *Harmonic Vibrations and Vibration Figures*. Newton and Co, London. [undated, was reviewed in *Nature* 1909]
 Ivins, W.M. (1938) *On the Rationalization of sight*. Papers, No. 8, Metropolitan Museum of Art, New York.
 Kelly, S. (2013) *Anthony McCall. 1970s Works on Paper*. Verlag der Buchhandlung Walther König.
 Poston, T. (1971a) *Fuzzy Geometry*, PhD Thesis, University of Warwick.
 Poston, T. (1971b) *Fuzzy Geometry, Manifold*, pp. 10, 25-33. <https://ianstewartjoat.weebly.com/manifold-10.html>
 Wagner, A. M. (2013) "Use values, or the line in time", in (Kelly, 2013) pp. 8-23.
 Whitehead, A.N. (1925) *An enquiry concerning the principles of natural knowledge*, second edition, Cambridge University Press.
 Whitehead, A.N. (1929) *Process and Reality: An Essay in Cosmology*, Cambridge University Press.